

Catalogue of Beurling's papers in the Uppsala University library

(The items marked by * are available from this website.)

Items of large size:

1. * Valda seminarieövningar i matematik Givna vid Uppsala Universitet från och med vårterminen 1938. Utskrivna av B. Nyman efter anteckningar av B. Kjellberg 332 pages, typed manuscript, in a folder.
2. * Resttermsproblemet vid så kallade Tauberteorem. Förberedande undersökning. 38 pages, xerox of typed manuscript. Ca. 1936.
Overlaps with Beurling's talk at the Scand. Math. Congress, 1938.
3. * Om några linära operationers framställning utanför sina konvergensområden 27 pages, xerox of typed manuscript. Ca 1936.
English translation with minor changes is included in CW-I, 420-429.
4. * Om kvasianalytiska konvergensthastigheter. Typed manuscript. Ca 1936. 25 pages.
With some changes incorporated in CW-I ("Mittag-Leffler lectures", Ch. II and III).
5. * Om Hardys sats rörande ζ -funktionens nollställen på $\sigma = \frac{1}{2}$, ca 1936, 11 pages, typed manuscript.
6. * Hardcover notebook I, 1948– 432 pages. Various subjects. Some pages are missing.
7. * Hardcover notebook II 185 pages, pp. 186–433 are empty. Various subjects.
8. * Hardcover notebook III, 433 pages. Pages 100–429 are empty. Various subjects.
9. Notes on Wiener Tauberian Theorem and the Prime Number Theorem Handwritten in Swedish. 10 pages, no page numbers.

10. Sur une nouvelle inégalites dans l'espace fonctionell L^p . Draft without page numbers. 8 pages.
Proof of Clarkson's inequalities.

Box Number 1

11. Distributioner. Note about analytic continuation and distributions. In Swedish. Handwritten, 7 pages.
12. Extremal length. Lecture at the Copenhagen congress (1946?) Unfinished manuscript. 8 pages.
13. No title. Topic: Orthogonality in the Hardy space H^∞ . Handwritten draft. 6 pages + earlier version?
14. In Swedish. No title. Contents: Positive definiteness of the kernel $f(x) + f(y) - f(x - y)$ and representation of functions in terms of Fourier-Stieltjes transforms of measures. 5 pages (computations appear on reverse sides).
Draft, the subject is close to “Sur quelques formes positives...”, CW-II, 71–86.
15. No title. Topic: A variational problem. Discussion of existence and estimates of minimizers. In Swedish. 5 pages.
16. Two handwritten manuscripts on entire functions, In French, pp. 6 + pp. 8. In 1977, Beurling dated them by 1927. Remarks on back side and on margins.
The first manuscript is entitled “Sur une propriété des fonctions entière avec de valeurs asymptotiques finies” It contains an answer to a question asked by Wiman in his lecture.
The second manuscript is related to the Denjoy conjecture. In the second manuscript page 7 and the end are missing.
17. No title. Contents: Notes about weighted shifts. 6 pages.
The content is included into the paper “A critical topology in harmonic analysis on semigroups”, CW-II, 285–298.
18. No title. Manuscript about pseudomeasures. In French. First page missing. 28 pages + 4 unnumbered pages.
Close to a series of Beurling’s C.R. notes included in CW-II.
19. No title. Two pages in English on the narrow topology.
The result contains in “Un Théorème sur les Fonctions Bornées et Uniformément Continues sur l’Axe Réel”, CW-II, 61–70.

20. No title. Six lectures on pseudomeasures. $5+5+5+5+4+4$ (preparatory lecture notes).
21. No title. Topic: Problems on integral equations and functional analysis. In Swedish. Intended for students. Typewritten, 3 pages.
22. Diverse problem. Various problems (in Swedish). Notes for the Uppsala seminar, 4 pages.
23. Om Selbergs bevis av primtalssatsen. Notes on Selberg's proof of the Prime Number Theorem. For the Uppsala seminar, 4 pages (in Swedish).
24. Om asymptotisk fas. On asymptotic phase. Scattering theory. For the Uppsala seminar, 4 pages (in Swedish).
25. En omordnings-sats av Polya. (A rearrangement theorem of Polya.) For the Uppsala seminar, 7 pages (in Swedish).
26. No title. On what is now called the Beurling transform. Notes in Swedish. For the Uppsala seminar, 4 pages.
27. Nollmängder för två klasser av analytiska funktioner. Relation of zero sets of the Bergman space and the space H^∞ in a domain. 4 pages in Swedish.
28. No title. Manuscript on univalent functions (in Swedish) 10 pages (pp 13–22).
29. No title. Notes on the Fourier-Carleman transform. 3 pages (in Swedish).
30. No title. Dirichlet series. Uniqueness. Besicovitch almost periodic functions. Carlson's theorem. $4 + 3$ pages in Swedish.
31. Linjära mängder. Additiva intervallfunktioner. 6 pages (in Swedish).
32. No title. Topic: Weighted convolution algebras (in Swedish). 7 pages.
33. No title. Notes related to the paper "An Automorphism of product measures". Notes for Uppsala seminar, 4 pages (in Swedish)

Box Number 2

34. * Black Notebook I. Softcover. Hela funktioner (V.T. 1944). Preparatory notes for a course (in Swedish). Notes pp 1–118, plus notes on pp. 181–184.
35. * Black notebook II. Softcover. Hela funktioner (H.T 1944). Preparatory notes for a course (in Swedish). Notes pp. 1–116. Uppsala, Sweden, Fall 1944.
36. * Black notebook III. Softcover. Funktionalanalys. Preparatory notes for a course (in Swedish). Notes pp. 1–242. Uppsala, Sweden, H.T. 1945.
37. Byproduct of the “balayage theory”. 4 pages.
Included in the paper “Local Harmonic Analysis with Some Applications to Differential Operators”, CW-II, 299–315.
38. No title. Topic: Densities related to Carlson’s theorem. 2 pages.
39. * No title Topic: Bilinear forms. Dirichlet spaces. 3 pages.
40. No title. Topic: Hypoellipticity through balayage (cf. Hörmander’s theorem), 6 pages.
Included in the paper “Local Harmonic Analysis with Some Applications to Differential Operators”, CW-II, 299–315.
41. Notes on Harmonic Analysis written for Deny, Jan 1963. Topic: Balayage and sampling. With application to negative definite functions, 5 pages.
The first 3.5 pages contain material from Beurling’s lectures on Balayage of Fourier-Stieltjes Transforms, CW-II, 341–350. The last 1.5 pages contain application to negative definite functions.
42. Dirichlet spaces. 27 pages. Marked **AC**.
Some pages contain computations with no explanations.
43. Harmonic analysis. Spectrum. Spectral synthesis, 14 pages. Draft. Marked **AF**.

Box Number 3

44. * Harmonic analysis. Lecture notes from lectures at Harvard (1948–49). (Notes taken by John Wermer.) 232 pages. One page written in Beurling’s handwriting is attached.
45. Stanford lecture notes 1–5. “On Quasianalyticity and General Distributions”. Full set.
46. Stanford lectures by P. Malliavin. Some applications of functions of exponential type to harmonic analysis.
47. Beurling lectures (semifinal form) Handwritten notes, Lectures 1, 2, 3, 4, 5, 6. Probably preliminary material for Stanford lectures. Marked **AD**.
48. Distributions. Handwritten. 9 pages. Marked **AD**.
This is Section IV of the paper “Analytic continuation across a linear boundary”, CW-I, 309–338.
49. No title. Handwritten note.
Close to section 9, “Analyticity and difference estimates”. CW-I, pp. 429–431. Typed version of the same, 2 pages.
Letter from Neuberger.
50. Idea to prove the Denjoy-Carleman theorem. Handwritten note, 2 pages.

Box number 4

51. Sur les Spectres des Fonctions. Typed version. Close to version published in Collected works, volume II, but with some additional handwritten remarks.
52. Soft Barrier case. Handwritten notes about free boundary problems, set I. 6+2 pages.
53. Existence proof (soft barrier) Handwritten notes about free boundary problems, set II. 23 pages.
54. The invariant boundary measure of an annulus. Not ordered. 16 pages.
Three different versions with some potential theory lemmas which Beurling needed for his work on the free boundary problem. The pages are not ordered.
55. * No title. Topic: Lemma on Fourier transforms of positive measures with applications to negative-definite functions. Ends with a conjecture. Handwritten, 3 pages.
56. * No title. Topic: Potential theory on discrete Abelian groups. Maximum principles. Handwritten note, 2 pages.
57. * No title. Topic: Dirichlet spaces. Handwritten note, 4 pages. The pages are not numbered.
Apparently, the first scanned page should be the last one
58. No title. Topic: An inequality related to convexity. In Swedish. 5 pages.

Box number 5

59. Beurling's lecture and discussions at Yeshiva University, 1963. Fourier transforms, balayage, applications to differential operators. 32 pages.

Main content was included in CW-II, 299–315.

60. Application of the extremal length to the free boundary-value problem. 21 pages.

Beurling-style material but it is not clear who was the author.

61. * Softcover notebook entitled: Anteckningar in Matematik (Problem, satser, förmodanden, mm.) 1936–1950. pp. 1–165, 200–202. Some pages are missing (that does not harm the text).

62. No title. Handwritten manuscript on entire functions of exponential type. Parts 2 and 3, 26 pages. Marked by **O**.

The content is close to “Some remarks on Entire Functions of exponential type”, CW-I, 386–395, but contains more results (mostly classical).

It seems that the handwriting is not Beurling's. It might be notes from Beurling's lectures.

Box number 6

63. * Lectures on the equation $\int_{-1}^1 \rho(t-s)f(s) ds = g_1(t)$, $|t| < 1$. Notes taken by Henry Pollak. 13 pages. Marked by **G**.
Cf. 117, 124.
64. Manuscript on majorant of functions of a real variable. Generalization of work of Beurling. Unknown author. 7 pages. Marked **V**.
65. No title. Handwritten, 7 pages. Marked **X**
Topics: A lemma on real functions. Finitely connected domains. Harmonic measure in finitely connected domains.
66. No title. Bell Labs lectures in Harmonic Analysis, ca. 1961. Marked **AA**.
Extremal problems related to Fourier transforms. Starts with Example 5.1.
67. * No title. Notes on discrete harmonic analysis with application to trigonometric polynomials. 9 pages. Marked **Y**.
68. * No title. Handwritten note on negative-definite functions 4 pages. Marked **U**.
69. * Quelques théorèmes que j'ai démontrés. Results on minimum modulus of analytic functions. 3 pages. Marked **R**.
70. Notes on Dirichlet Series, 13 pages. Marked **S** Handwritten draft of the paper with the same title, CW-II 221-228.
71. * No title. New York University lectures on convexity and Banach spaces February 1956. 53 pages (35 pages of lecture notes + some drafts). Marked **E**.

Box number 7

72. * Beurling, A. & Hewitt, E., An approximation method in non-compact spaces, with some application to harmonic analysis. Typed manuscript. 27 pages.

The folder also contains a handwritten manuscript with same title and another version with corrections.

73. A theorem on functions defined on a semi-group. Typed manuscript. 7 pages. Marked **M**.

Preliminary version of a published paper, CW-II, 157–161.

74. No title. Topic: Weak convergence of subharmonic functions with application to entire functions of exponential type. Handwritten manuscript, 4 pages. Marked **L**.

75. No title. Topic: Gabriel's problem and Blaschke products. Handwritten manuscript. 25 pages. Marked **N**.

A version of Beurling's last paper "On interpolation and Blaschke products", CW-I, 339–354. Differs from the published version.

76. * Additive functions of intervals. Notes taken by L. Ehrenpreis. Theory of finite additive measure on \mathbb{R} with applications to harmonic analysis. Not published. 51 pages. Marked **P**.

The pages were wrongly numbered. Related to Beurling's approach to pseudomeasures; likely, they predate his C.R. Notes, CW-II.

77. Schrödinger problem. Material related to "An Automorphism of Product Measures". Lectures at Bell labs. 1–25 pages. Marked **AB**.

78. No title. Lectures on Wiener's tauberian theorem. Notes taken by Henrik Martens. Lectures at Bell labs, 26–34 pages. Marked **AB**.

79. The minimal problem and the Schrödinger equation. Handwritten manuscript. 15 pages, the pages are not numbered.

Related to the paper "An Automorphism of Product Measures", CW-II, 209–220.

80. The minimal problem for $B(\theta)$. Handwritten manuscript. 8 pages, the pages are not numbered
Related to the paper “An Automorphism of Product Measures”, CW-II, 209–220.

Box number 8.

Dirichlet spaces

Joint project with Deny.

81. Letter from Deny, May 25, 1956, 2 pages.
Comments to items 99-103, which are preliminary versions of the Acta (1958) paper by Beurling and Deny, CW-II.
82. Potential Theory. Notes taken from Deny's lectures at London Math. Soc. 1961. 35 pages
The last section contains a discussion of Beurling's ideas.
83. * On the ratio of two negative definite functions. By Arne Beurling and Jacques Deny. Typed manuscript intended for publication, pp. 1-27. With letter from Deny of Jan 6, 1964. 1 page + Deny's comment to paper, 1 page.
The main results of this paper appeared in Kahane's Bourbaki seminar talk, but in the special case when $G = \mathbb{Z}$.
84. Espaces de Dirichlet -I - Le cas élémentaire. By A. Beurling and J. Deny, 18 pages.
Typed manuscript of the paper published in Acta Math. 1958. CW-II.
85. Dirichlet spaces. By A. Beurling and J. Deny.
Manuscript of the paper in Proc. Nat. Acad. Sci. USA, 1959, 10 pages (CW-II).
86. Formes et espaces de Dirichlet. Typed manuscript. 26 pages.
A version of Deny's talk at the Bourbaki Seminar, Expo no. 187, 1959/1960. Available at the NUMDAM archive.
87. Notebook named Dirichlet Spaces. Handwritten.
Mostly contains drafts and computations.
88. Maximum principle and duality in potential theory Manuscript, 4 pages.
Likely, it is written by Deny.

89. Spectral synthesis and maximum principle, 4 pages
Likely, it is written by Deny.
90. Gauss-Poincaré spaces, 6 pages.
A short extract from this note was published by Deny in French: “Espaces de Gauss-Poincaré et espaces de Dirichlet”. 1964. Séminaire de Théorie du Potentiel, dirigé par M. Brelot, G. Choquet et J. Deny, 7e année; 1962/63, No. 4, 2 pages.
(available at the NUMDAM archive).
91. Dirichlet spaces whose all elements are continuous. Original manuscript and xeroxed with 1 attached page 2 pages + handwritten comments, 1 page.
Items 91-97 (as well as 88, 89, and 104) contain proofs of results announced in two papers by Beurling-Deny.
Likely, all of them were written by Deny.
92. Conformal mapping on Dirichlet spaces. 4 pages handwritten + 1 typed page
93. A formula for the differential part of the norm, 2 pages, unnumbered.
94. Majorization for $\int_E |u|^\alpha dx$, 2 pages.
95. Regular functions with positive kernels and maximum principle, 2 pages.
96. Spectre d'un élément d'un espace de Dirichlet. Synthèse spectrale, 4 pages
97. Some remarks on the algebra of continuous multipliers, 2 pages.
98. Espaces de Dirichlet - I - Généralités (project de Note aux Comptes-Rendus) 3 pages.
Announcement of results that appear in the Acta paper (1958) by Beurling-Deny.
99. Manuscript: Chapitre I. (Etat no 2.) Espaces de Dirichlet général, 9 pages.

The items 99-103 are preliminary versions of the Acta (1958) paper by Beurling and Deny. It seems that they contain more material comparing with the published version.

100. Manuscript: Chapitre I. Espaces de Dirichlet Généraux. 15 pages.
101. Manuscript: Chapitre I. Définitions générales et premières propriétés 13 pages.
102. Manuscript: Chapitre II. Théorie du potentiel dans les espaces de Dirichlet réguliers 14 pages.
103. Manuscript: Chapitre III. Espaces de Dirichlet spécial, 8 pages.
104. 2 - Le spectre d'un élément d'un espace de Dirichlet régulier pp. 13-17.
See comment to the item 91.
105. One xeroxed page related to Dirichlet spaces.
Xerox of page 6 from the item 103.
106. Espaces de Dirichlet sur un ensemble fini. Typed manuscript, 5 pages.
Proof of results related to the example from the Acta (1958) paper.
107. Fonctions définies négatives. Typed manuscript, 15 pages. Marked **H**.
Items 107-108 contain proofs of basic properties of negative definite functions. In the manuscript prepared for publication in Acta (item 83), most of these properties were mentioned without proof.
108. Fonctions définies négatives — (cas symétrique). Typed manuscript. 11 pages. Marked **K**.

Box number 9

109. No title. Manuscript on locally compact Abelian groups, 5 pages.
Not clear who is the author.
110. Notebook L^p -parallelogram olikheter. Geometry in Banach spaces.
Fall term 1960 87 pages. Marked **Z**.
Mostly drafts and computations. Likely, preparatory notes for a course, cf. item 71.
111. Notebook: Helmholtz problem in potential theory.
Mostly drafts and computations.
112. A brief sketch of the S -integral. Lemmas on conjugate functions.
16 pages. Marked **T**
Apparently, consists of two different parts, probably written in different times and not related to each other.
Pages 1–8: Extension of the Stieltjes integration (related to the Dirichlet spaces, and to Beurling's approach to pseudomeasures).
Pages 9–16: Lemmas on conjugate functions (similar to the ones used in the work with Malliavin).
In the end of the second part written "Not finished".
113. No title. Handwritten manuscript 12 pages, unnumbered.
Preliminary draft related to the item 72.
On the corner of the first page it is written "E.Hewitt, early September".
114. No title. Notes named "Whitney" about sums of angles in polygons. 2 pages.
115. No title. Watson problem. Decay of analytic functions. Dated 20/1 44. 3 pages.

Box number 10

116. No title. Topic: Wiener Tauberian theorem. Handwritten notes, 9 pages, marked **C**.
117. * The Integral Equation $\int_0^\infty \rho(t-s)f(s) ds = \lambda f(t)$, $t > 0$. The Spaces A_1^2 and B_1^2 . Typewritten manuscript, 30 pages. Marked **D**.
118. * Seminarieövningar i matematik. Givna vid Uppsala Universitet från och med höstterminen 1949. Continuation of item 1. Typewritten manuscript.
119. * Seminarieövningar i matematik. Givna vid Uppsala Universitet från och med höstterminen 1950. Continuation of item 1 and item 118. Typewritten manuscript.
120. * Seminarieövningar i matematik Givna vid Uppsala Universitet från och med höstterminen 1951 och vårterminen 1952 Continuation of items 1, 118 and 119. Typewritten manuscript.
121. No title. Three draft manuscripts related to Beurling-Malliavin theory. Typed, pp. 29–39; handwritten, pp. 4; handwritten 12–30. Four letters from Malliavin.
122. Sur une nouvelle inégalité dans l'espace fonctionnel L^p , Typed manuscript. 6 pages.
Similar to item 10.
123. No title. Topic: Density of linear combinations of exponentials, Handwritten manuscript. 3 pages.
124. * No title. Topic: Wiener-Hopf equation on the real axis (lecture notes). Handwritten, 4 pages.
125. Continuous Dirichlet kernels. Handwritten notes, 2 pages.
126. Généralisation d'une inégalité de Cauchy. Nordiska Matematisk Tidskrift (1932) 2 pages.
Inductive proof of Hölder's inequality for n functions. The first Beurling publication.

127. Spektralproblemet. In Swedish. Handwritten, 2 pages.
128. No title. Introduction to balayage. Lectures notes. Typed manuscript, 6 pages.
129. No title. Handwritten notes on a question in the additive number theory, and on Dirichlet spaces, 10 pages.
Mostly, some computations.
130. No title. Draft related to the paper “Sur les fonctions limites quasianalytiques des fractions rationnelles”, CW-I.
In Swedish. Handwritten, 5 pages.
131. Beurling seminar (likely at Princeton, ca. 1960) Lectures by Beurling VI, VII, VIII, IX, X (not organized, writings by pencil on back side), XI (not organized, writings by pencil on back side), XIII, XIV, another version of lecture XIV, not organized. (Lectures VI and VII resp. VIII and IX are stapled together.)
Förteckning över teorem och lemmata i Beurlings föreläsningar 1959–60. (Catalog of theorems and lemmas in Beurling’s lectures 1959–1960.)
132. Draft related to the paper “Analytic Continuation across a linear boundary”, CW-I. pp. 39–51.
133. Seminar notes 1963, number 2, 3, 4, 5, 6.
Topic: Beurling’s paper “A critical topology in harmonic analysis on semigroups”, CW-II.